9.2 Comparing Inscribed Angles

YOU WILL NEED

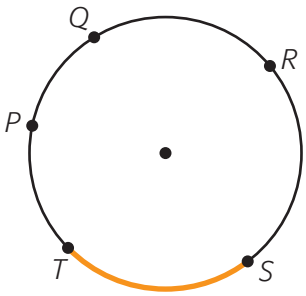
- geometry software
  or
- a compass
- a protractor
- a ruler

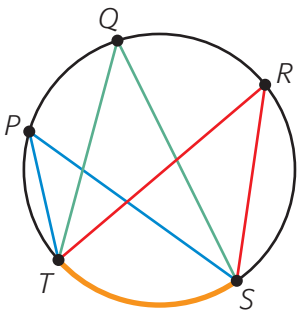
GOAL

Relate the measures of inscribed angles in a circle.

INVESTIGATE the Math

Francis is making a dreamcatcher for his young sister. The first part of his design is shown.





? What is the relationship among the inscribed angles in Francis's design?

A. Draw a circle. Define a minor arc by placing points S and T on the circle. Place three points, P, Q, and R on the major arc.

B. Draw the three inscribed angles ($\angle P$,$\angle Q$, and $\angle R$) subtended by minor arc ST. Use a different colour for each angle as shown at left. Measure these angles. What do you notice?

C. Place point P at a different location on major arc TS. What happens to the measure of $\angle P$?

D. Repeat step C for points Q and R. Does your observation for point P also hold for points Q and R?

E. Repeat steps A to D using a different minor arc ST.

F. Summarize the relationship among the inscribed angles in Francis's design.

Reflecting

G. How are the measures of inscribed $\angle P$, $\angle Q$, and $\angle R$ related to the central angle subtended by minor arc ST?

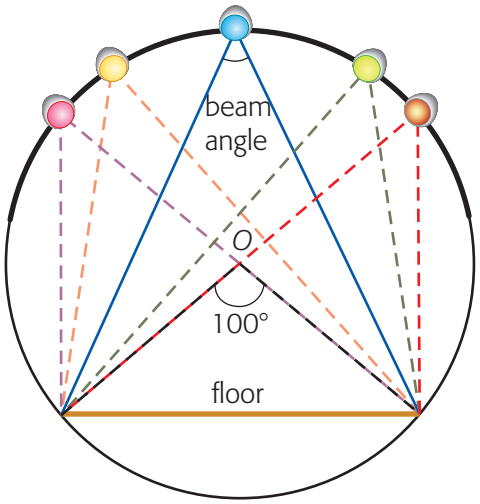
H. Why does changing the location of points P, Q, and R not change the measure of the inscribed angle?

I. Explain why changing the size of minor arc ST causes the measure of the inscribed angles to change but remain equal.

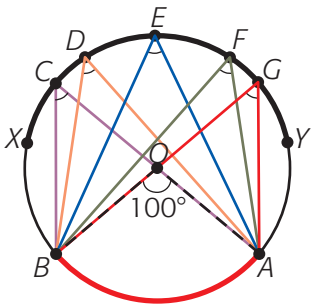
WORK WITH the Math

EXAMPLE 1 | Determining the measure of inscribed angles

The lighting engineers for a circus are hanging spotlights on tubing shaped in an arc as shown. They want each light to illuminate the floor area. The tubing lies on a minor arc of the circle centred at O. At what beam angle setting should each spotlight be set to fully illuminate the floor?



Rani's Solution



I used my drawing program to represent the stage, arc, and lights as a circle.
I chose A and B to represent the ends of the floor and X and Y to represent the ends of the tubing arc.
I named the places where the lights go as points C to G.
I knew that points C, D, E, F, and G all lie on the circle centred at O.

I needed to know the measure of the inscribed angles.

$\angle BOA = 100°$

$\angle C = \left(\frac{1}{2}\right)\angle BOA$

$= \frac{1}{2} \times 100°$

$= 50°$

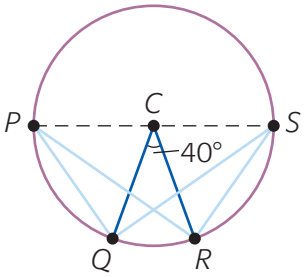
$\angle C = \angle D = \angle E = \angle F = \angle G = 50°$

$\angle BOA$ is a central angle subtended by minor arc AB. $\angle C$, $\angle D$, $\angle E$, $\angle F$, and $\angle G$ are all inscribed angles subtended by minor arc AB. So I knew these inscribed angles were all the same measure, which is half the central angle.

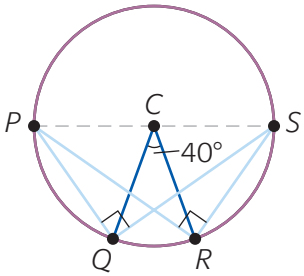
The beam angle setting for each spotlight should be 50°.

EXAMPLE 2 | Determining missing angles

A magician is designing a logo for his business. His logo is drawn in a circle centred at C. What are the measures of $\angle QPR$, $\angle PQS$, $\angle PRS$, and $\angle QSR$ in the logo?



Zachary's Solution



I copied the logo.
I knew that PS is a diameter because it goes through the centre of the circle, so arc PS is a semicircle. This means that $\angle PQS$ and $\angle PRS$ are inscribed angles subtended by a semicircle, so they are each 90°.

$\angle PQS = \angle PRS = 90°$

$\angle QPR = \left(\frac{1}{2}\right)\angle QCR$

$= \frac{1}{2} \times 40°$

$= 20°$

$\angle QPR = \angle QSR = 20°$

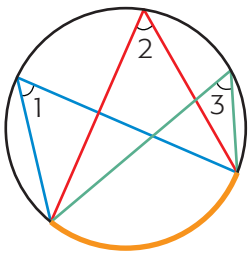
I noticed that $\angle QPR$ and $\angle QSR$ are inscribed angles subtended by minor arc QR. This minor arc also subtends central $\angle QCR$. So $\angle QPR$ and $\angle QSR$ are half of $\angle QCR$.

In the logo, $\angle QPR = \angle QSR = 20°$ and $\angle PQS = \angle PRS = 90°$.

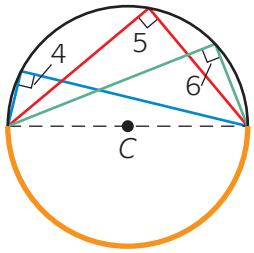
In Summary

Key Idea

- It is possible to have many inscribed angles subtended by the same arc. Angles 1, 2, and 3 have the same measure. If the arc is a semicircle, the inscribed angles are 90°.



$\angle 1 = \angle 2 = \angle 3$

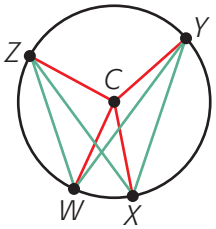


$\angle 4 = \angle 5 = \angle 6 = 90°$

Checking

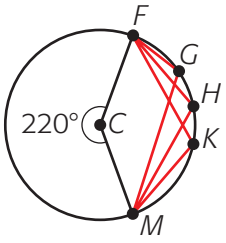
1. For each circle with centre C, determine the measure of the inscribed angles indicated.

a)



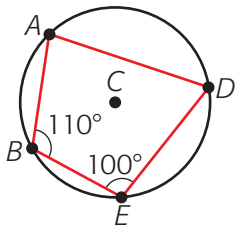
$\angle WCX = 36°$,
$\angle ZCX = 110°$
$\angle ZWY = ?$,
$\angle WYX = ?$

b)



$\angle G = ?$, $\angle H = ?$,
$\angle K = ?$

c)

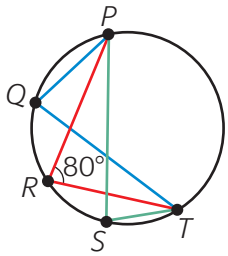


$\angle A = ?$, $\angle D = ?$

Practising

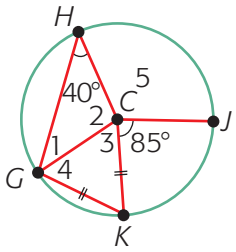
2. Determine the unknown angle measures indicated. Show your work.

a)



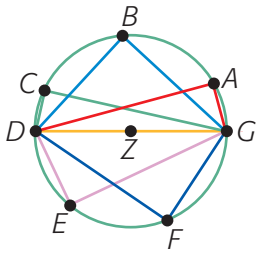
$\angle Q$ and $\angle S$

b)



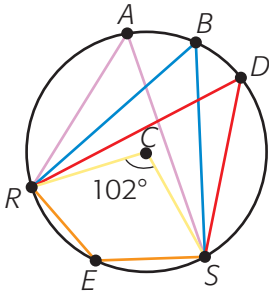
$\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, and $\angle K$

c)



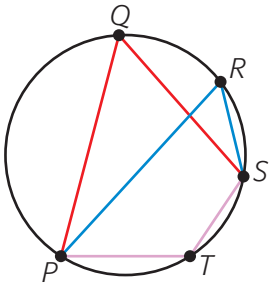
DG is a diameter.
$\angle A$, $\angle B$, $\angle C$, $\angle E$, and $\angle F$

3. Determine the measures of $\angle A$, $\angle B$, $\angle D$, and $\angle E$ shown at right. Explain how you determined the measure of each angle.

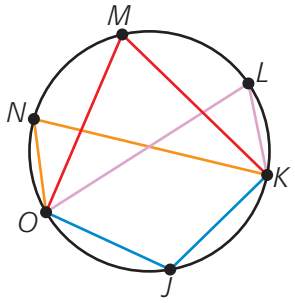


4. In each diagram, state which inscribed angles are equal. Explain how you know.

a)

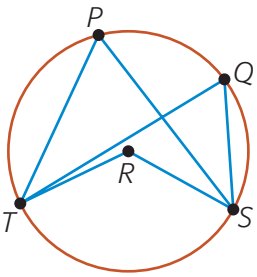


b)



5. **Multiple choice.** R is the centre of the circle. Which statement is *false*?

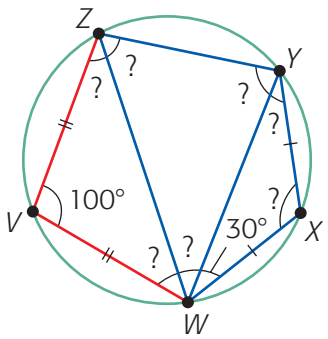
A. $\angle P = \angle Q$
B. $\angle P$ may be greater than 90°.
C. $\angle P$ and $\angle Q$ will always be less than $\angle R$.
D. $\angle P$, $\angle Q$, and $\angle R$ are all subtended by arc ST.



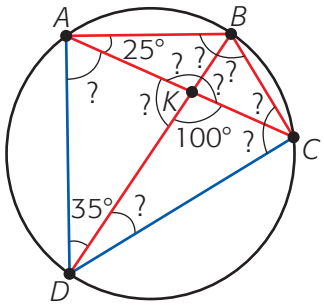
6. Draw a circle. Describe how to inscribe a rectangle in the circle without using a protractor or geometry software.

7. Determine the measures of the angles indicated. Show your work.

a)



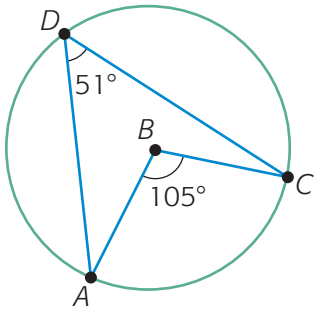
b)



8. Construct a circle with centre O. Then draw any inscribed quadrilateral $PQRS$.
   a) Measure each angle of the quadrilateral.
   b) Determine the sum of opposite angles ($\angle P + \angle R$ and $\angle Q + \angle S$). What do you notice?
   c) Change the shape of the quadrilateral by moving one or more of the vertices to a different position on the circle. Examine the sums of each pair of opposite angles. What do you notice?
   d) Make a **conjecture** about opposite angles in a quadrilateral inscribed in a circle.
   e) Draw radii OS and OQ. What is the measure of each central angle? Can you explain why your conjecture works?

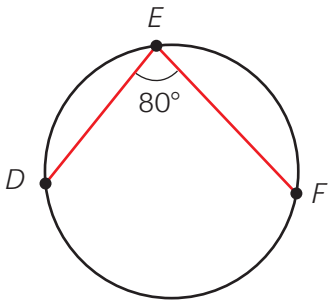
9. Create a question involving two or more inscribed angles. Write a solution to your question. Exchange questions with a classmate and solve each other's questions.

10. Is point B the centre of the circle at left? Explain how you know.



Closing

11. How does knowing that $\angle DEF = 80°$ help you to draw other 80° angles without a protractor?



Extending

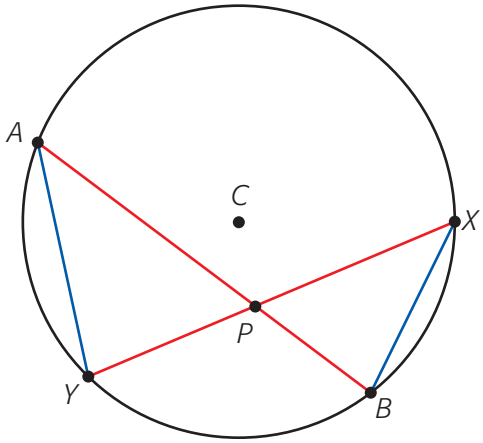
12. Rani claims that, if she randomly places four points at different locations on a circle, she can always create four pairs of equal inscribed angles. Is she correct? Explain.

13. Two chords, AB and XY, intersect at P as shown. The centre of the circle is at C.

a) State all pairs of equal angles.

b) What do you know about $\triangle APY$ and $\triangle XPB$?

c) Explain why $(PB)(PA) = (PX)(PY)$.



14. In a circle with centre at M, $\angle PRS = 85°$. Determine the measures of all angles indicated in quadrilateral $PRST$.