9.3 Chord Properties

YOU WILL NEED

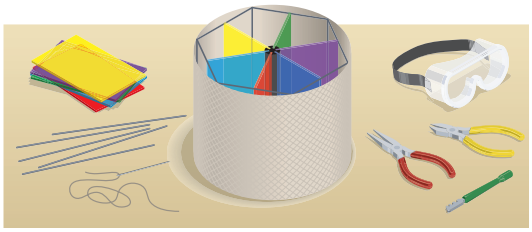
- geometry software
  or
- a compass
- a protractor
- a ruler

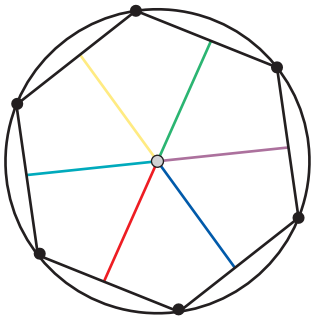
GOAL

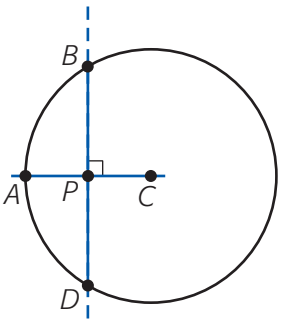
Examine the relationship between the centre of a circle and chords.

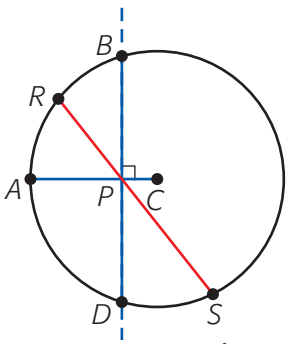
INVESTIGATE the Math

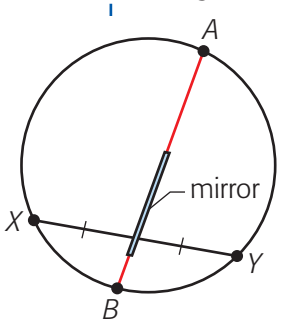
Nola designs lamps. The shade of one of her lamps has six different coloured glass inserts, which are connected to a wire frame. Each insert needs to be perpendicular to the frame. A **cross-section** of the shade is shown.











? How can Nola check that the glass inserts are connected correctly to the frame?

A. Draw a circle with centre C and radius CA. Place point P anywhere on radius CA.

B. Draw a line perpendicular to the radius through P as shown at left. Label the points where the line intersects the circle as B and D.

C. Measure segments PB and PD. How do their measures compare?

D. Move point P to a different location on radius CA and repeat step C. Does your answer to part C depend on where P is on the radius?

E. Draw another chord RS through P, but not perpendicular to radius CA. Measure segments PR and PS. Does the relationship you discovered in part C still hold?

F. At what point on each chord of the wire frame should the glass insert be connected to ensure that it is perpendicular to the frame?

Reflecting

G. If you held a mirror as shown, how could you tell if line segment AB goes through the centre of the circle?

H. What conclusions can you draw about the angle created by joining the centre of a circle to the midpoint of a chord?

EXAMPLE 1 | Verifying A Chord Property

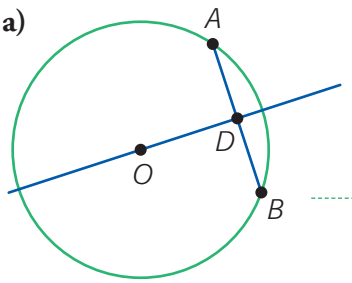
Show that:

a) A line drawn from the centre of a circle that is perpendicular to a chord, bisects the chord.

b) The perpendicular bisector of a chord passes through the centre of the circle.

Luc's Solution

a)

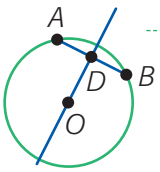


AB = 3.40 cm
AD = 1.70 cm
BD = 1.70 cm
m∠ODA = 90.00°

I used geometry software to construct a circle. I constructed two points on the circle and joined them to create chord *AB*. I selected the chord and the centre of the circle and constructed a perpendicular line. Then I located the point of intersection between the two lines and labeled this point *D*. I measured the lengths of *AB*, *AD*, and *BD*. I also measured ∠*ODB*.

D is the midpoint of *AB*, since *AD* = *BD*.

The line through *OD* is perpendicular to *AB* and bisects the chord.



AB = 1.70 cm
AD = 0.85 cm
BD = 0.85 cm
m∠ODA = 90.00°

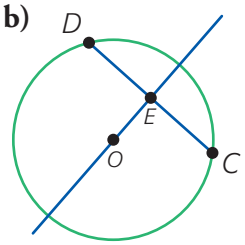
I repeated this process for a different circle and a different chord. The result was the same.

D is the midpoint of *AB*, since *AD* = *BD*

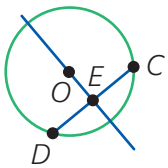
The perpendicular line from the centre of a circle to a chord bisects the chord.

Every time I repeated this process using a different circle and a different chord, the result was the same.

b)



I used geometry software to construct a circle. I constructed two points on the circle and joined them to create chord *CD*. Then I constructed the midpoint of the chord, *E*. I selected the chord and the midpoint and constructed a perpendicular line. This line passed through the centre of the circle.



I repeated this process for a different circle and a different chord. The result was the same.

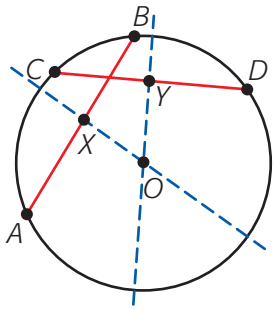
The perpendicular bisector of a chord passes through the centre of a circle.

Every time I repeated this process using a different circle and a different chord the result was the same.

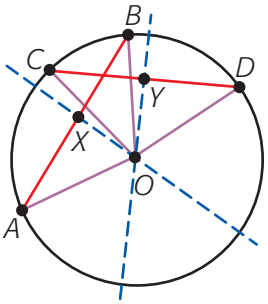
EXAMPLE 2 | Determining which chords can locate the centre of a circle

Is it possible to locate the centre of a circle using any two chords?

Erin's Solution: Trying with non-parallel chords



I decided to try to locate the centre of a circle using two chords, *AB* and *CD*, that are not parallel to each other. I constructed the perpendicular bisectors of each chord. I used perpendicular bisectors since I know they pass through the centre of the circle. I reasoned that their point of intersection, *O*, might be the centre of the circle.



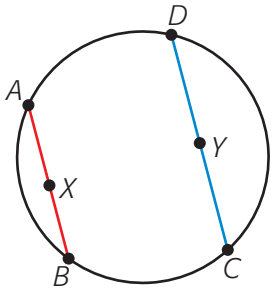
I drew line segments *OA*, *OB*, *OC*, and *OD*. Then I measured them. They were all equal.
Since a circle is the set of points that are the same distance from a centre point, *O* must be the centre of the circle.

$OA = 3$ cm $\quad OC = 3$ cm
$OB = 3$ cm $\quad OD = 3$ cm

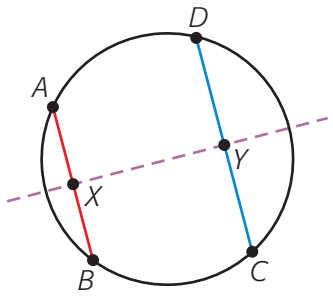
Check:
I put the point of my compass at *O* and the pencil on *D*, and drew a circle. The circle went through *A*, *B*, and *C*, so *O* is the centre of the circle.

The centre of a circle is at the intersection of the perpendicular bisectors of any pair of non-parallel chords.

Zachary's Solution: Trying with parallel chords



I wondered if it was possible to locate the centre of a circle if the chords were parallel.
I drew a circle with two parallel chords, *AB* and *CD*.
I constructed the midpoints, *X* and *Y*, of each chord.



I constructed a perpendicular line through *X*. I noticed that the perpendicular bisector of *AB* went through *Y*. Then I constructed a perpendicular line through *Y*. The perpendicular bisector of *CD* went through *X*.

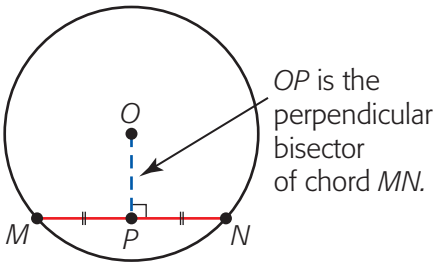
If two chords are parallel, you cannot locate the centre of a circle directly using only their perpendicular bisectors.

Since both perpendicular bisectors are the same line, I could not determine the location of the centre.
I know the centre is on line segment *XY*, but I don't know where.

In Summary

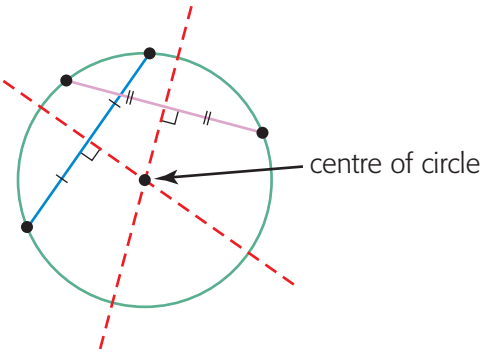
Key Idea

- A line that passes through the centre of a circle and the midpoint of a chord is perpendicular to the chord. Another way of saying this is that a line that is perpendicular to a chord and also passes through the centre of the circle bisects the chord.

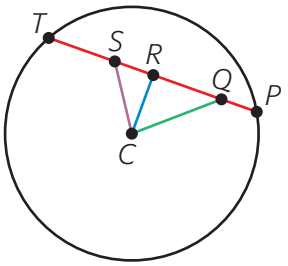


Need to Know

- The perpendicular bisector of a chord passes through the centre of a circle.
- The centre of a circle is located at the intersection of the perpendicular bisectors of two non-parallel chords.



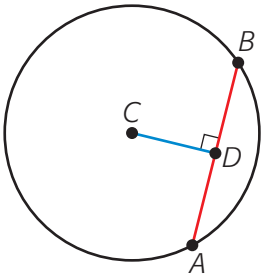
Checking



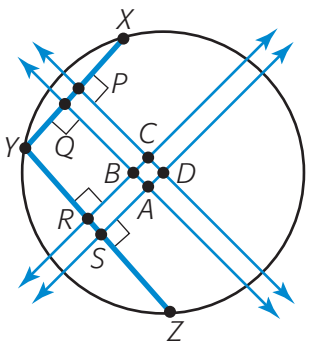
1. In the diagram, $RP = RT$. State which angles, if any, are 90°. Explain how you know.

2. Erin wants to place an umbrella holder at the centre of a circular sandbox. How can Erin locate the correct position for the umbrella holder?

Practising

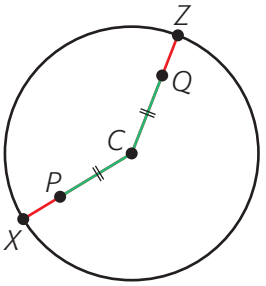


3. **Multiple choice.** In the circle shown, $AB = 12$ cm. Which statement is true?
   - **A.** The radius is 12 cm.
   - **B.** $AD = 6$ cm
   - **C.** $CD = 12$ cm
   - **D.** $CD = AB$

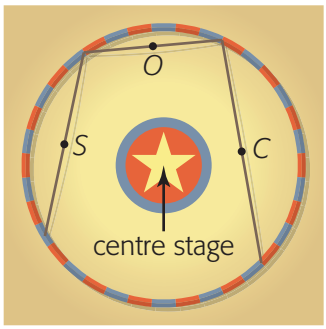


4. **Multiple choice.** In the diagram, $PX = PY$ and $SY = SZ$. Which point is at the centre of the circle?
   A. A **B.** B **C.** C **D.** D

5. **a)** Is it possible to locate the centre of a circle using two non-parallel chords if one of the chords is a diameter? Use a diagram to show why or why not.
   b) What would happen if both chords were diameters?

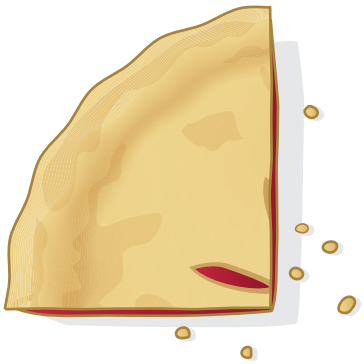


6. **a)** Draw a circle with radii and points as shown.
   b) Draw chords through P and Q, perpendicular to each radius. Measure the chords. What do you notice?
   c) Make a **conjecture** based on what you discovered in part b).
   d) Use reflecting properties to determine if your conjecture will work for any position of P and Q when $CP = CQ$.

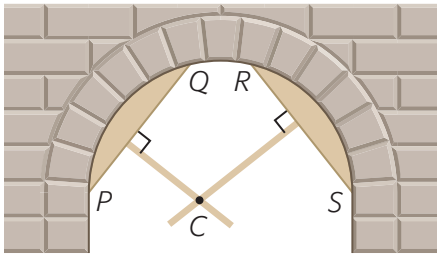


7. Three students have bought tickets to attend a performance of Le Cirque du Soleil. They are sitting at points S, O, and C as shown at left. Use chord properties to determine if their seats are equidistant from centre stage.

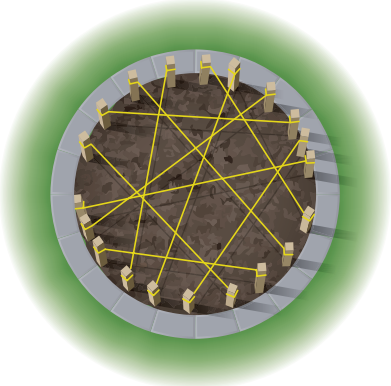
8. Luc goes to his favourite bakery, where pies are $12.00 each. He orders one fourth of a pie for $3.00, and is given the wedge shown. Even though the angle is 90°, he wonders if he has been short-changed. Is the wedge less than, greater than, or really one fourth of a pie?



9. A carpenter's apprentice has been asked to determine the centre of the circular part of an arch, and provides the explanation below. Do you agree with his thinking? Why or why not?

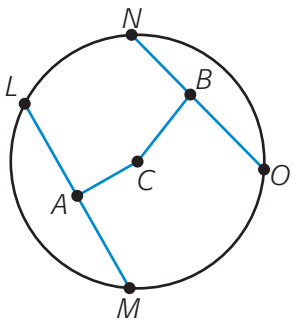


First, I attached two planks, *PQ* and *RS*, to the ceiling. I nailed two pieces of wood perpendicular to each plank. Since a line perpendicular to a chord goes through the centre of a circle, the intersection of the two pieces of wood must be the centre of the circle.



10. A landscape architect has staked out a circular garden as shown. She has marked one diameter in the soil for the gardener to plant his first row of flowers. However, children playing in the garden have added more lines. Explain how, using chord properties, the gardener could determine which chord is the diameter.

11. Show that a circle can be drawn through any three points that do not lie on a straight line. Is it possible to draw more than one circle through these points? Explain, using chord properties.



Closing

12. Explain how you can use chord properties to determine which chord is longer and which angles are right angles.

Extending

13. Two fragments of plates were found during an archeological dig. Is it possible that these fragments came from the same plate? Explain.

14. **a)** Copy the diagram below. Draw chord *PR*. What do you know about this chord?
    b) Use chords to locate the centre, *S*, of the circle.
    c) Label the midpoints of *PQ* and *QR* as *T* and *V*. Draw quadrilateral *STQV*. What kind of a quadrilateral is it? How do you know?